

Controlled Hawking Process by Quantum Energy Teleportation

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Abstract

In this paper, a new quantum mechanical method to extract energy from black holes with contracting horizons is proposed. The method is based on a gedanken experiment on quantum energy teleportation (QET), which has been recently proposed in quantum information theory. We consider this QET protocol for N massless fields in near-horizon regions of large-mass black holes with near-horizon geometry described by the Minkowski metric. For each field, a two-level spin is strongly coupled with the local quantum fluctuation outside the horizon during a short time period. After the measurement of N fields, N -bit information is obtained. During the measurement, positive-energy wave packets of the fields form and then fall into the black hole. The amount of excitation energy is independent of the measurement result. After absorption of the wave packets and increase of the black hole mass, a measurement-result-dependent local operation of the N fields is performed outside the horizon. Then, accompanying the extraction of positive energy from the quantum fluctuation by the operation, negative-energy wave packets of the fields form and then fall into the black hole, decreasing the black hole mass. This implies that a part of the absorbed positive energy emitted from the measurement devices is effectively retrieved from the black hole via the measurement results.

1 Introduction

Essentially, two methods are known by which energy can be extracted from black holes. One is the classical method that is based on the existence of ergospheres of rotating black holes. It includes the Penrose mechanism of test particles [1], the superradiance of an electromagnetic field, [2] and the Blandford-Znajek effect of accretion disks with magnetic field [3] for Kerr black holes. Moreover, it is possible to extract energy from Reissner-Nordstrom charged black holes because ergosphere-like structures of the black holes appear for charged particles [4]. Even though energy can be extracted from black holes using these classical methods, the horizon areas of the black holes do not decrease. If the quantum effect of matter is taken into account, energy extraction with a decrease in horizon area is possible. This is the second method of energy extraction, called the Hawking effect [5], which describes black hole evaporation. Black holes emit thermal radiation, accompanied by generation of negative energy flux that falls into the black holes. The Hawking effect takes place spontaneously outside the horizon and cannot be stopped artificially. In this paper, a new quantum mechanical method of energy extraction from black holes is proposed. This method is based on a gedanken experiment on quantum energy teleportation (QET), which has been recently proposed in quantum information theory [6]–[10]. In this method, energy is extracted by a local quantum operation after a quantum measurement of quantum fluctuation of fields outside the horizons; thus, it is not a spontaneous phenomenon like the Hawking process. Besides, the time scale of the QET process is very short compared to the Hawking-process time scale of order of inverse of black-hole temperature.

The protocols of QET consist of local operations and classical communication. By measuring the local fluctuation induced by a zero-point oscillation in the ground state of a many-body quantum system and by announcing the measurement result to distant points, energy can be effectively teleported without breaking any physical laws including causality and local energy conservation. The key point is that there exists quantum correlation between local fluctuations of different points in the ground state. Therefore, the measurement results of local fluctuation in a region include information about fluctuations in other regions. By selecting and performing a proper local

operation that is based on the announced information, the local zero-point oscillation of a place far from the measurement point can be squeezed, yielding negative energy density. During the local operation, surplus energy of the local fluctuation is emitted to the external systems and can be harnessed.

We consider this QET protocol for N massless fields in the near-horizon regions of large-mass black holes with near-horizon geometry described by the Minkowski metric. The Hartle–Hawking state of the fields in the black hole spacetime, which is the thermal equilibrium state with the Hawking temperature [11], is reduced to the Minkowski vacuum state in flat spacetime. For each field, a two-level spin is strongly coupled with the local quantum fluctuation outside the horizon during a short time period. The spin is measured to give 1-bit information about the fluctuation. After the measurement of N fields, N -bit information, denoted by D_k ($k = 1 \sim 2^N$), is obtained. The post-measurement quantum states outputting D_k are not the vacuum states but the states excited by the measurement devices. Positive-energy wave packets of the fields form and then fall into the black hole. The amount of excitation energy is independent of D_k . After absorption of the wave packets and increase in the black hole mass, a D_k -dependent local operation of the N fields is performed outside the horizon. Then, along with the extraction of positive energy from the fluctuation by the operation, negative-energy wave packets of the fields form and then fall into the black hole. The mass of the black hole decreases due to absorption of the negative flux. This means that a part of the absorbed positive energy emitted from the measurement devices is effectively retrieved from the black hole via the measurement results. Though energy extracted by QET does not exhibit thermal property like Hawking radiation, the energy extraction process involving measurement and the local operation, which generates a pair of excitations with positive and negative energy, is regarded as a controlled Hawking process. It is noteworthy that if a unitary operation corresponding to a different N -bit sequence $D_{k' \neq k}$ is performed, the amount of energy extracted by QET decreases. Without any information about the correct D_k , a unitary operation corresponding to a random N -bit sequence does not extract energy from the fields, but gives energy to the fields on average. This implies that a black hole, which swallows the wave packets generated by the measurement with output D_k , remembers the information about D_k . Thus, black holes absorbing quantum states with different D_k are physically different even though their classical geometries are the same.

The paper is organized as follows. In section 2, we briefly review the

horizon shift of large-mass black holes in classical theory, using the CGHS model. In section 3, we analyze a gedanken experiment on QET in a near-horizon region. The summary and discussion are provided in section 4. We adopt the natural unit $c = \hbar = 1$.

2 Horizon Shift in Classical Gravity

To briefly review the property of horizon shift in classical gravity, we adopt a two-dimensional soluble dilaton gravity model called the CGHS model [12] for simplicity. For higher-dimensional spherically symmetric black holes with large masses, we obtain essentially the same conclusion by treating falling matter with a spherically symmetric distribution. In the CGHS model, the action is given by

$$S = \int d^2x \sqrt{-g} \left(e^{-2\varphi} (R + 4(\nabla\varphi)^2 + 4\Lambda^2) - \frac{1}{2} \sum_{n=1}^N (\nabla f_n)^2 \right),$$

where φ is a real scalar dilaton field, Λ is a positive constant, and f_n denotes N real massless matter fields. The static black hole solutions are given by

$$ds^2 = - \left(1 - \frac{M}{\Lambda} e^{-2\Lambda r} \right) d\tau^2 + \left(1 - \frac{M}{\Lambda} e^{-2\Lambda r} \right)^{-1} dr^2,$$

where M represents the black hole mass. The horizon radius is given by $r_H = \frac{1}{2\Lambda} \ln \left(\frac{M}{\Lambda} \right)$. Using new coordinates defined by $x^\pm = \pm \frac{1}{\sqrt{M\Lambda}} \sqrt{e^{2\Lambda r} - \frac{M}{\Lambda}} e^{\pm \Lambda \tau}$, the metric is rewritten as

$$ds^2 = - \frac{dx^+ dx^-}{1 - \Lambda^2 x^+ x^-}.$$

The future horizon stays at $x^- = 0$ in the new coordinates. Because the scalar curvature on the horizon is proportional to Λ^2 , a very small Λ yields the Minkowski spacetime as the near-horizon geometry. In the near-horizon region with $|x^\pm| \ll 1/\Lambda$, the metric is reduced to $ds^2 = -dx^+ dx^-$. The matter field f_n obeys the equation $\partial_+ \partial_- f_n = 0$ in the region and can be decomposed into left- and right-moving solutions as follows: $f_n = f_{Ln}(x^+) + f_{Rn}(x^-)$. Solutions with falling matter into the black hole with mass M are also analytically obtained [12] as

$$ds^2 = -\frac{dx^+ dx^-}{1 - \Lambda^2 x^+ x^- - \frac{\Lambda}{2M} \int_{-\infty}^{x^+} dy^+ \int_{-\infty}^{y^+} dz^+ T_{++}(z^+)},$$

where $T_{++}(x^+)$ is the incoming energy flux and is given by $\sum_{n=1}^N (\partial_+ f_{Ln}(x^+))^2$. Therefore, the metrics of general solutions in the x^\pm coordinates also become the flat metric $ds^2 = -dx^+ dx^-$ when $M \gg \Lambda$ and $|x^\pm| \ll 1/\Lambda$. Hence, the horizon shift due to absorption of matter cannot be observed in these coordinates of the near-horizon region.

In order to observe the horizon shift, rescaled coordinates are available; these are given by $X^\pm = \sqrt{\frac{M}{\Lambda}} x^\pm$ [12]. The metric is then rewritten as

$$ds^2 = -\frac{dX^+ dX^-}{\frac{M}{\Lambda} - \Lambda^2 X^+ X^- - \frac{1}{2} \int_{-\infty}^{X^+} dY^+ \int_{-\infty}^{Y^+} dZ^+ T_{++}(\sqrt{\frac{\Lambda}{M}} Z^+)}. \quad (1)$$

From Eq. (1), the horizon shift is completed soon after the falling matter passes. If the support of $T_{++}(x^+)$ is given by $(-\infty, x_s^+)$, the horizon stops soon after $X_s^+ = \sqrt{\frac{M}{\Lambda}} x_s^+$. The metric in the region with $X^+ > X_s^+$ is given by

$$ds^2 = -\frac{dX^+ dX^-}{\frac{M'}{\Lambda} - \Lambda^2 X^+ (X^- - X_H)},$$

where M' is the enlarged black hole mass and the shifted future horizon stays at $X^- = X_H = -\frac{1}{2\Lambda^2} \sqrt{\frac{M}{\Lambda}} \int_{-\infty}^{x_s^+} T_{++}(x^+) dx^+$. Even for higher dimensional spherically symmetric black holes of Einstein gravity, the horizon also stabilizes soon after the falling matter passes if the matter takes the shape of spherical shells. This is because any spherically symmetric vacuum solution of the Einstein equation is static and equal to the Schwarzschild solution. After the horizon is shifted, the classical geometry of the black holes holds no information except the total energy. However, as explained in the next section, the black hole remembers more information quantum mechanically.

3 Quantum Energy Teleportation

The horizon of the initial black hole stays at $x^- = 0$. Let us consider N quantum fields \hat{f}_n ($n = 1 \sim N$) in the near-horizon region with the x^\pm

coordinates. Because the fields are N copies of a massless scalar field, we can concentrate on one of the fields and disregard the index n for the time being. We consider a QET process with a time scale much shorter than inverse of the black hole mass which is the time scale of generation of the Hawking radiation. Thus, the Hawking process can be neglected in the later discussion. The equation of motion is written as $\partial_+ \partial_- \hat{f} = 0$, and the general solution is solved by the quantum left- and right-moving solutions as $\hat{f} = \hat{f}_+(x^+) + \hat{f}_-(x^-)$. The spacetime coordinates of the Minkowski metric are given by $t = \frac{1}{2}(x^+ + x^-)$ and $x = \frac{1}{2}(x^+ - x^-)$. The canonical conjugate momentum operator of $\hat{f}(x) = \hat{f}|_{t=0}$ is defined by $\hat{\Pi}(x) = \partial_t \hat{f}|_{t=0}$ and it satisfies the standard commutation relation, $[\hat{f}(x), \hat{\Pi}(x')] = i\delta(x - x')$. The left-moving wave $\hat{f}_+(x^+)$ can be expanded in terms of plane-wave modes as

$$\hat{f}_+(x^+) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left[\hat{a}_\omega^L e^{-i\omega x^+} + \hat{a}_\omega^{L\dagger} e^{i\omega x^+} \right],$$

where \hat{a}_ω^L ($\hat{a}_\omega^{L\dagger}$) is an annihilation (creation) operator of a left-moving particle and satisfies

$$[\hat{a}_\omega^L, \hat{a}_{\omega'}^{L\dagger}] = \delta(\omega - \omega'). \quad (2)$$

The right-moving wave $\hat{f}_-(x^-)$ can be also expanded in the same way using the plane-wave modes. The energy density operator is given by

$$\hat{\varepsilon}(x) = \frac{1}{2} : \hat{\Pi}(x)^2 : + \frac{1}{2} : \left(\partial_x \hat{f}(x) \right)^2 :,$$

where $::$ denotes the normal order of creation-annihilation operators for the plain-wave modes. The Hamiltonian is given by $\hat{H} = \int_{-\infty}^\infty \hat{\varepsilon}(x) dx$. In the near-horizon region, the Hartle–Hawking state [11] is described by the Minkowski vacuum state $|0\rangle$, defined by $\hat{H}|0\rangle = 0$. It is also satisfied that $\langle 0 | \hat{\varepsilon}(x) | 0 \rangle = 0$. Let us define the chiral momentum operators as

$$\hat{\Pi}_\pm(x) = \hat{\Pi}(x) \pm \partial_x \hat{f}(x).$$

Then, the energy density can be rewritten as

$$\hat{\varepsilon}(x) = \frac{1}{4} : \hat{\Pi}_+(x)^2 : + \frac{1}{4} : \hat{\Pi}_-(x)^2 :. \quad (3)$$

Following the method given in [6], we perform QET for the state $|0\rangle$ as follows. Figure 1 shows a spacetime diagram that expresses this QET experiment. Consider a probe system P of a two-level spin located in a small compact region $[x_1, x_2]$ satisfying $x_1 > 0$ outside the horizon in order to detect fluctuations of \hat{f} . In a similar way of particle-detector interaction of Unruh [16], we introduce a measurement Hamiltonian between \hat{f} and the spin probe such that

$$\hat{H}_m(t) = g(t)\hat{G}_+ \otimes \hat{\sigma}_y,$$

where $g(t)$ is a time-dependent real coupling constant, \hat{G}_+ is given by

$$\hat{G}_+ = \frac{\pi}{4} + \int_{-\infty}^{\infty} \lambda(x) \hat{\Pi}_+(x) dx, \quad (4)$$

$\lambda(x)$ is a real function with support $[x_1, x_2]$, and $\hat{\sigma}_y$ is the y -component of the Pauli matrices of the probe spin. We assume that the initial state of the probe is the up state $|+\frac{1}{2}\rangle$ of the z -component $\hat{\sigma}_z$. The time dependence of $g(t)$ is assumed to be generated by a device with an external energy supply from spatial infinity, respecting local energy conservation. In the later analysis, we choose a sudden switching form such that $g(t) = \delta(t - 0)$. After the interaction is switched off, we measure the z -component $\hat{\sigma}_z$ for the probe spin. If the up or down state, $|+\frac{1}{2}\rangle$ or $|-\frac{1}{2}\rangle$, of $\hat{\sigma}_z$ is observed, we assign the bit number $b = 0$ or 1 , respectively, to the measurement result. The measurement is completed at $t = +0$. The time evolution of this measurement process with output b can be described by the measurement operators \hat{M}_b , which are often used in quantum information theory [13] and which satisfy

$$\hat{M}_b \rho \hat{M}_b^\dagger = \text{Tr}_P \left[\left(I \otimes \left| (-1)^b \frac{1}{2} \right\rangle \left\langle (-1)^b \frac{1}{2} \right| \right) \hat{V} \left(\rho \otimes \left| +\frac{1}{2} \right\rangle \left\langle +\frac{1}{2} \right| \right) \hat{V}^\dagger \right],$$

where ρ is an arbitrary density operator of the field, the time evolution operator $\hat{V} = \text{T exp} \left[-i \int_{-0}^t \hat{H}_m(t') dt' \right]$ is computed as $\exp \left[-i \hat{G}_+ \otimes \hat{\sigma}_y \right]$ for $t > 0$, and the trace Tr_P is taken to the probe system. The measurement operators \hat{M}_b are evaluated as

$$\hat{M}_b = \left\langle (-1)^b \frac{1}{2} \right| \exp \left[-i \hat{G}_+ \otimes \hat{\sigma}_y \right] \left| +\frac{1}{2} \right\rangle.$$

Hence, we obtain the explicit expression of \hat{M}_b such that

$$\hat{M}_0 = \cos \hat{G}_+, \quad (5)$$

$$\hat{M}_1 = \sin \hat{G}_+. \quad (6)$$

For the vacuum state $|0\rangle$, the probability of obtaining b in the measurement $\langle 0|\hat{M}_b^\dagger \hat{M}_b|0\rangle$ is independent of b and is given by $1/2$. The explicit calculation of $\langle 0|\hat{M}_b^\dagger \hat{M}_b|0\rangle$ is as follows.

$$\begin{aligned} \langle 0|\hat{M}_b^\dagger \hat{M}_b|0\rangle &= \langle 0|\left[\frac{1}{2} + \frac{(-1)^b}{2} \cos(2\hat{G}_+)\right]|0\rangle \\ &= \frac{1}{2} - \frac{(-1)^b}{2} \langle 0|\sin\left(2 \int_{-\infty}^{\infty} \lambda(x) \hat{\Pi}_+(x) dx\right)|0\rangle \\ &= \frac{1}{2}. \end{aligned}$$

In the above calculation, $\sin\left(2 \int_{-\infty}^{\infty} \lambda(x) \hat{\Pi}_+(x) dx\right)$ is calculated as the sum of the products of odd-number field operators and the correlation functions with odd-number field operators vanish for the vacuum state $|0\rangle$ of this free field. The post-measurement states of \hat{f} for the result b are calculated as

$$|\psi_b\rangle = \sqrt{2}\hat{M}_b|0\rangle = \frac{i^b}{\sqrt{2}} \left(e^{-\frac{\pi}{4}i} |\lambda\rangle + (-1)^b e^{\frac{\pi}{4}i} |-\lambda\rangle \right), \quad (7)$$

where $|\pm\lambda\rangle$ are left-moving coherent states defined by

$$|\pm\lambda\rangle = \exp\left[\pm i \int_{x_1}^{x_2} \lambda(x) \hat{\Pi}_+(x) dx\right] |0\rangle. \quad (8)$$

The two states $|\psi_0\rangle$ and $|\psi_1\rangle$ are non-orthogonal to each other with $\langle\psi_0|\psi_1\rangle = \langle\lambda|-\lambda\rangle \neq 0$ because the measurement is not projective [13]. For convenience, let us introduce the Fourier transformation of $\lambda(x)$ as follows.

$$\tilde{\lambda}(\omega) = \int_{-\infty}^{\infty} \lambda(x) e^{i\omega x} dx, \quad (9)$$

where the coefficient satisfies $\tilde{\lambda}^*(-\omega) = \tilde{\lambda}(\omega)$. Note that the left-moving momentum operator $\hat{\Pi}_+(x^+)$ can be expanded as

$$\hat{\Pi}_+(x^+) = -i \int_0^\infty d\omega \sqrt{\frac{\omega}{\pi}} \left[\hat{a}_\omega^L e^{-i\omega x^+} - \hat{a}_\omega^{L\dagger} e^{i\omega x^+} \right]. \quad (10)$$

Using Eqs. (2), (9), and (10), we obtain the following relation:

$$\exp \left[-i \int_{x_1}^{x_2} \lambda(x) \hat{\Pi}_+(x) dx \right] \hat{a}_\omega^L \exp \left[i \int_{x_1}^{x_2} \lambda(x) \hat{\Pi}_+(x) dx \right] = \hat{a}_\omega^L - \sqrt{\frac{\omega}{\pi}} \tilde{\lambda}(\omega). \quad (11)$$

Therefore, the coherent state in Eq. (8) satisfies

$$\hat{a}_\omega^L |\pm \lambda\rangle = \mp \sqrt{\frac{\omega}{\pi}} \tilde{\lambda}(\omega) |\pm \lambda\rangle, \quad (12)$$

which is derived from Eq. (11) and $\hat{a}_\omega^L |0\rangle = 0$. Using $a_\omega^{L\dagger}$, the state $|\pm \lambda\rangle$ can be expressed as

$$|\pm \lambda\rangle = \exp \left[-\frac{1}{2} \int_0^\infty d\omega \frac{\omega}{\pi} |\tilde{\lambda}(\omega)|^2 \right] \exp \left[\mp \int_0^\infty d\omega \sqrt{\frac{\omega}{\pi}} \tilde{\lambda}(\omega) a_\omega^{L\dagger} \right] |0\rangle.$$

From this expression, we get the following inner products:

$$\langle \lambda | -\lambda \rangle = \langle -\lambda | \lambda \rangle = \langle 0 | 2\lambda \rangle = \exp \left[-2 \int_0^\infty d\omega \frac{\omega}{\pi} |\tilde{\lambda}(\omega)|^2 \right]. \quad (13)$$

The expectational value of the Heisenberg operator of energy density $\hat{\varepsilon}(x, t)$ for the post-measurement state in Eq. (7) is given by

$$\begin{aligned} \langle \psi_b | \hat{\varepsilon}(x, t) | \psi_b \rangle &= \frac{1}{2} [\langle \lambda | \hat{\varepsilon}(x, t) | \lambda \rangle + \langle -\lambda | \hat{\varepsilon}(x, t) | -\lambda \rangle] \\ &\quad - (-1)^b \text{Im} \langle \lambda | \hat{\varepsilon}(x, t) | -\lambda \rangle. \end{aligned} \quad (14)$$

From Eq. (12), it is easily checked that $\text{Im} \langle \lambda | \hat{\varepsilon}(x, t) | -\lambda \rangle$ vanishes. In fact, we can show

$$\begin{aligned} &\text{Im} \langle \lambda | \hat{\varepsilon}(x, t) | -\lambda \rangle \\ &= -\text{Im} \left[\int_0^\infty d\omega \frac{\omega}{\pi} \left[\tilde{\lambda}(\omega) e^{-i\omega x^+} + \tilde{\lambda}(\omega)^* e^{i\omega x^+} \right] \right]^2 \\ &= 0. \end{aligned}$$

Besides, Eqs. (3), (12), and (10) yield the following relations:

$$\langle \lambda | \hat{\varepsilon}(x, t) | \lambda \rangle = \langle -\lambda | \hat{\varepsilon}(x, t) | -\lambda \rangle = (\partial_+ \lambda(x^+))^2.$$

Substituting the above equations into Eq. (14), we can conclude that the measurement excites wave packets propagating to the horizon with an energy density that does not depend on b such that

$$\langle \psi_b | \hat{\varepsilon}(x, t) | \psi_b \rangle = (\partial_+ \lambda(x^+))^2. \quad (15)$$

This result independent of the measurement result b is striking as compared to a particle detector model in the Rindler spacetime discussed by Unruh and Wald. In the reference [17], a particle detector coupled with a thermal bath of a quantum field is analyzed which excites the field with positive energy when the detector makes a transition and is observed in an excited state. If the state of the detector is observed in the initial ground state, we have different excitation of the field. Especially, in a causally disconnected region, negative energy is induced by the detector. Thus amount of energy induced by the detector in the model has explicit dependence of the measurement results. However, in our POVM measurement model, the probe spin as a detector excites the quantum field independent of the measurement result as seen in Eq. (15). Moreover, soon after the measurement, the energy density for each post-measurement state vanishes except around the measurement point. The amount of total average excitation energy is evaluated as

$$E_A = \int_{-\infty}^{\infty} (\partial_+ \lambda(x))^2 dx. \quad (16)$$

The energy flux falls into the horizon at $x^- = 0$. Let us assume that at $t = T$, the left-moving excitation has already been eliminated from $[x_1, x_2]$ and satisfies $\lambda(x + T) = 0$ for $x \in [x_1, x_2]$. The average state at time T is expressed as

$$\hat{\rho}_M = \sum_b e^{-iT\hat{H}} \hat{M}_b |0\rangle \langle 0| \hat{M}_b^\dagger e^{iT\hat{H}}.$$

It is worth noting that the state $\hat{\rho}_M$ is a strictly localized state defined by Knight [14], because $\hat{\rho}_M$ is locally the same as $|0\rangle\langle 0|$ at $t = T$ and satisfies $\text{Tr}[\hat{\rho}_M \hat{\varepsilon}(x)] = 0$ for $x \in [x_1, x_2]$. Moreover, for each state $|\psi_b\rangle$, $2n$ -point functions of the left-moving operators $\hat{\Pi}_+$ with integer n are the same as those of $|0\rangle$ outside the horizon after time T . Let us consider a $2n$ -point

function $\langle \psi_b | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \psi_b \rangle$ and assume that all x_k satisfy $x_k > 0$ and that they do not coincide with each other. Then, using Eq. (7), the function is given by

$$\begin{aligned} & \langle \psi_b | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \psi_b \rangle \\ &= \frac{1}{2} \left[\langle \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \lambda \rangle \right. \\ & \quad \left. + \langle -\lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | -\lambda \rangle \right] \\ & \quad - (-1)^b \text{Im} \langle \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | -\lambda \rangle. \end{aligned} \quad (17)$$

Here, we have used the relation

$$\begin{aligned} & \langle \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | -\lambda \rangle^* \\ &= \langle -\lambda | \hat{\Pi}_+(x_{2n} + T) \cdots \hat{\Pi}_+(x_1 + T) | \lambda \rangle \\ &= \langle -\lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \lambda \rangle, \end{aligned}$$

which is ensured by the commutation relation

$$[\hat{\Pi}_+(x), \hat{\Pi}_+(y)] = 2i\delta'(x - y), \quad (18)$$

and the fact that all $x_k + T$ do not coincide with each other. Because \hat{f} is a free field, we can adopt the Wick's theorem for the composite operator $\hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T)$ as

$$\begin{aligned} & \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) \\ &= \sum : \hat{\Pi}_+(x_{s_1} + T) \cdots \hat{\Pi}_+(x_{s_{2k}} + T) : \\ & \quad \times \prod \langle 0 | \hat{\Pi}_+(x_{w_1} + T) \hat{\Pi}_+(x_{w_2} + T) | 0 \rangle, \end{aligned} \quad (19)$$

where the contraction sum runs over only those contractions between operators in different normal-ordered products. Then, using Eq. (19) and the fact that $\lambda(x_k + T) = 0$, we get

$$\langle \pm \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \pm \lambda \rangle = \langle 0 | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | 0 \rangle. \quad (20)$$

This is because the normal ordered contributions $\langle \pm \lambda | : \hat{\Pi}_+(x_{s_1} + T) \cdots \hat{\Pi}_+(x_{s_{2k}} + T) : | \pm \lambda \rangle$ vanish:

$$\begin{aligned} & \langle \pm \lambda | : \hat{\Pi}_+(x_{s_1} + T) \cdots \hat{\Pi}_+(x_{s_{2k}} + T) : | \pm \lambda \rangle \\ &= 2^{2k} \partial_x \lambda(x_{s_1} + T) \cdots \partial_x \lambda(x_{s_{2k}} + T) = 0. \end{aligned}$$

Thus, $\langle \pm \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \pm \lambda \rangle$ is reduced into a sum of products of n two-point functions for the ground state $|0\rangle$ and equal to $\langle 0 | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | 0 \rangle$:

$$\begin{aligned} & \langle \pm \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \pm \lambda \rangle \\ &= \sum \langle 0 | \hat{\Pi}_+(x_{w_1} + T) \hat{\Pi}_+(x_{w_2} + T) | 0 \rangle \cdots \langle 0 | \hat{\Pi}_+(x_{w_{2n-1}} + T) \hat{\Pi}_+(x_{w_{2n}} + T) | 0 \rangle \\ &= \langle 0 | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | 0 \rangle. \end{aligned}$$

By use of Eq. (19), it can be also proven that

$$\text{Im} \langle \lambda | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | -\lambda \rangle = 0. \quad (21)$$

This is because the following relation holds:

$$\begin{aligned} & \langle \lambda | : \hat{\Pi}_+(x_{s_1} + T) \cdots \hat{\Pi}_+(x_{s_{2k}} + T) : | -\lambda \rangle \\ &= (-1)^k Q(x_{s_1} + T) \cdots Q(x_{s_{2k}} + T) \langle \lambda | -\lambda \rangle, \end{aligned}$$

where $Q(x + T)$ is a real function defined by

$$Q(x + T) = - \int_0^\infty d\omega \frac{\omega}{\pi} \left[\tilde{\lambda}(\omega) e^{-i\omega(x+T)} + \tilde{\lambda}(\omega)^* e^{i\omega(x+T)} \right],$$

and $\langle \lambda | -\lambda \rangle$ is also real as seen in Eq. (13). Moreover, taking into account the fact that all x_k do not coincide with each other, it can be observed that the two-point functions $\langle 0 | \hat{\Pi}_+(x_{w_1} + T) \hat{\Pi}_+(x_{w_2} + T) | 0 \rangle$ are real. Thus, by using the above results and Eq. (19), we obtain Eq. (21). By substitution of Eqs. (20) and (21) into Eq. (17), it is proven that the $2n$ -point function in the outside region after time T is the same as that of the vacuum state:

$$\begin{aligned} & \langle \psi_b | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | \psi_b \rangle \\ &= \langle 0 | \hat{\Pi}_+(x_1 + T) \cdots \hat{\Pi}_+(x_{2n} + T) | 0 \rangle. \end{aligned}$$

It is worth noting that many-point functions of the energy density operator $\hat{\varepsilon}(x, T)$ can be obtained from these $2n$ -point functions with the point-splitting regularization scheme [15]. Besides the left-moving mode, the many-point functions of the right-moving mode for $|\psi_b\rangle$ are the same as those for $|0\rangle$. Therefore, after the wave packets are absorbed by the black holes, the fluctuations in energy density for the state $|\psi_b\rangle$ are the same as those of the vacuum $|0\rangle$:

$$\langle \psi_b | \hat{\varepsilon}(x_1, T) \cdots \hat{\varepsilon}(x_n, T) | \psi_b \rangle = \langle 0 | \hat{\varepsilon}(x_1, T) \cdots \hat{\varepsilon}(x_n, T) | 0 \rangle.$$

Because the massless field can affect the spacetimes of black holes only via energy density, the difference in the measurement result b yields no difference in black-hole spacetimes. The difference in b appears only in odd-number many-point functions of $\hat{\Pi}_+$. For example, the one-point function of $\hat{\Pi}_+$ for $|\psi_b\rangle$ is dependent on b and is given by

$$\langle \psi_b | \hat{\Pi}_+(x+T) | \psi_b \rangle = (-1)^b \int_0^\infty d\omega \frac{\omega}{\pi} \left[\tilde{\lambda}(\omega) e^{-i\omega(x+T)} + \tilde{\lambda}(\omega)^* e^{i\omega(x+T)} \right]. \quad (22)$$

We recall here that we have N copies of the field \hat{f} . Thus, the measurement output bit ($b_n = 0$ or 1) is obtained for each field \hat{f}_n and the total data of the measurements are recorded as $D_k = (b_1 b_2 \cdots b_N)$ with $k = 1 \sim 2^N$. If we take a large N , quantum fluctuation in the total energy density of the N fields is severely suppressed due to the law of large numbers, and the back reaction of the quantum fields to the black hole can be treated by a classical gravity equation of the metric. In the equation, the average energy flux as a source term is given by $N(\partial_+ \lambda(x^+))^2$. The falling flux yields a horizon shift of δX_H^- independent of b in the X^\pm coordinates as $\delta X_H^- = -\frac{N}{2\lambda^2} \sqrt{\frac{M}{\lambda}} \int_{-\infty}^\infty (\partial_+ \lambda(x^+))^2 dx^+$.

Figure 2 shows the horizon shift. The initial horizon ($X^- = 0$) is denoted by H_I in the figure. The new horizon after the shift is denoted by H_F . It should be noted that a spacetime region becomes static soon after the flux passes through the region. If we do nothing more, H_F is the final event horizon. However, as seen below, if the unitary operation dependent on D_k is performed, negative energy flux is created and the horizon shifts again. The receding horizon is denoted by H_{QET} . The argument does not change at all if the falling energy flux interacts with collapsing matter within the black hole. In Figure 2, right-moving matter represented by dashed lines collides with the falling positive-energy flux inside the horizon.

At $t = T$, we perform a unitary operation on the quantum field \hat{f} ; the unitary operation is dependent on b and is given by

$$\hat{U}_b = \exp \left[i\theta (-1)^b \int_{-\infty}^\infty p(x) \hat{\Pi}_+(x) dx \right], \quad (23)$$

where θ is a real parameter fixed below and $p(x)$ is a real function with the same support of $\lambda(x)$, $[x_1, x_2]$. We do not require any classical communication

phase for this black-hole QET because the operation in Eq. (23) is performed in the same region $[x_1, x_2]$ as that where the measurement is executed. After the operation, the average state of the field \hat{f} is given by

$$\hat{\rho}_F = \sum_{b=0,1} \hat{U}_b e^{-iT\hat{H}} \hat{M}_b |0\rangle \langle 0| \hat{M}_b^\dagger e^{iT\hat{H}} \hat{U}_b^\dagger.$$

Let us introduce an energy operator localized around the region $[x_1, x_2]$ such that $\hat{H}_B = \int_{-\infty}^{\infty} w(x) \hat{\varepsilon}(x) dx$. Here, $w(x)$ is a real window function with $w(x) = 1$ for $x \in [x_1, x_2]$ and it rapidly decreases outside the region. The average amount of energy around the region is evaluated as

$$\begin{aligned} E_B &= \text{Tr} [\hat{\rho}_F \hat{H}_B] \\ &= \sum_{b=0,1} \langle 0| \hat{M}_b^\dagger e^{iT\hat{H}} \hat{U}_b^\dagger \hat{H}_B \hat{U}_b e^{-iT\hat{H}} \hat{M}_b |0\rangle. \end{aligned}$$

In order to simplify the expression of E_B , a useful relation is available:

$$\hat{U}_b^\dagger \hat{H}_B \hat{U}_b = \hat{H}_B - \theta(-1)^b \int_{x_1}^{x_2} \partial_x p(x) \hat{\Pi}_+(x) dx + \theta^2 \int_{x_1}^{x_2} (\partial_x p(x))^2 dx.$$

This is derived from Eqs. (18) and (23). Thus, E_B is rewritten as

$$\begin{aligned} E_B &= \sum_b \langle 0| \hat{M}_b^\dagger e^{iT\hat{H}} \hat{H}_B e^{-iT\hat{H}} \hat{M}_b |0\rangle \\ &\quad - \theta \sum_b (-1)^b \langle 0| \hat{M}_b^\dagger \int_{x_1}^{x_2} \partial_x p(x) \hat{\Pi}_+(x+T) dx \hat{M}_b |0\rangle \\ &\quad + \theta^2 \int (\partial_x p(x))^2 dx. \end{aligned} \tag{24}$$

Because we assume that $T > |x_2 - x_1|$, the following two relations can be directly proven:

$$\left[e^{iT\hat{H}} \hat{H}_B e^{-iT\hat{H}}, \hat{M}_b \right] = 0, \tag{25}$$

$$\left[\int_{x_1}^{x_2} \partial_x p(x) \hat{\Pi}_+(x+T) dx, \hat{M}_b \right] = 0. \tag{26}$$

From Eqs. (5) and (6), we are able to check the following relations:

$$\sum_b \hat{M}_b^\dagger \hat{M}_b = 1, \quad (27)$$

$$\sum_b (-1)^b \hat{M}_b^\dagger \hat{M}_b = \cos(2\hat{G}_+). \quad (28)$$

Substituting Eqs. (25)–(28) into Eq. (24), we obtain the following expression of E_B :

$$\begin{aligned} E_B &= \langle 0 | e^{iT\hat{H}} \hat{H}_B e^{-iT\hat{H}} | 0 \rangle \\ &\quad - \theta \int_{x_1}^{x_2} dx \partial_x p(x) \langle 0 | \hat{\Pi}_+(x+T) \cos(2\hat{G}_+) | 0 \rangle \\ &\quad + \theta^2 \int (\partial_x p(x))^2 dx. \end{aligned}$$

Note that

$$\langle 0 | e^{iT\hat{H}} \hat{H}_B e^{-iT\hat{H}} | 0 \rangle = \langle 0 | \hat{H}_B | 0 \rangle = \int_{-\infty}^{\infty} w(x) \langle 0 | \hat{\varepsilon}(x) | 0 \rangle dx = 0.$$

Therefore, the average energy $E_B = \text{Tr} [\hat{\rho}_F \hat{H}_B]$ after the operation is computed as

$$E_B = -\theta\eta + \theta^2\xi,$$

where $\xi = \int_{x_1}^{x_2} (\partial_x p(x))^2 dx$ and

$$\eta = \int dx \partial_x p(x) \langle 0 | \hat{\Pi}_+(x+T) \cos(2\hat{G}_+) | 0 \rangle. \quad (29)$$

It is possible to simplify this expression of η . Using Eqs. (10) and (12) and the relation given by

$$\cos(2\hat{G}_+) | 0 \rangle = \frac{i}{2} (| +2\lambda \rangle - | -2\lambda \rangle),$$

we obtain the following relation:

$$\langle 0 | \hat{\Pi}_+(x+T) \cos(2\hat{G}_+) | 0 \rangle = -\frac{2}{\pi} \langle 0 | 2\lambda \rangle \int_0^\infty d\omega \omega \tilde{\lambda}(\omega) e^{-i\omega(x+T)}.$$

Substituting Eq. (9) into the above relation, we obtain

$$\begin{aligned} & \langle 0 | \hat{\Pi}_+(x+T) \cos(2\hat{G}_+) | 0 \rangle \\ &= -\frac{2}{\pi} \langle 0 | 2\lambda \rangle \int_0^\infty dy \lambda(y) \frac{1}{(x-y+T)^2}. \end{aligned} \quad (30)$$

By substituting Eq. (30) into Eq. (29), we get the final expression of η as

$$\eta = -\frac{4}{\pi} |\langle 0 | 2\lambda \rangle| \int_{x_1}^{x_2} \int_{x_1}^{x_2} p(x) \frac{1}{(x-y+T)^3} \lambda(y) dx dy. \quad (31)$$

By fixing the parameter θ such that

$$\theta = \frac{\eta}{2\xi}$$

so as to minimize E_B , it is proven that the average energy around $[x_1, x_2]$ of \hat{f} takes a negative value, that is,

$$E_B = -\frac{\eta^2}{4\xi} < 0. \quad (32)$$

By virtue of local energy conservation, this result implies that the unitary operation extracts positive energy $+|E_B|$ from the field to the external systems. Taking into account the existence of N fields, the total amount of extracted energy is $N|E_B|$. The measurement step for $D_k = (101 \cdots 0)$ is shown in Figure 3. In Figure 4, the extraction step is depicted. This process is analogous to the standard Hawking process of black holes, that is, pair creation of particles with positive and negative energy outside the horizon [5]. Just like in the Hawking process, the created negative energy $-N|E_B|$ of the fields falls into the horizon and decreases the black hole mass, accompanying the second shift of the horizon ($H_F \rightarrow H_{QET}$), as shown in Figure 2. For each field, the sum of the positive energy generated by the measurement and the negative energy generated by the operation is always positive, that is, $E_A + E_B > 0$. This means that $E_A > |E_B|$, and thus only a part of the absorbed positive energy E_A can be retrieved from a black hole by this QET process.

When we consider higher-dimensional spherical black holes, essentially the same results are obtained. The POVM measurement and operation for energy extraction are performed non-locally over a spherical-shell region surrounding an event horizon with a fixed radius.

4 Summary and Discussion

In this paper, a new quantum mechanical method to extract energy from black holes with contracting horizons is proposed. This quantum mechanical method is based on a gedanken experiment on quantum energy teleportation (QET), which has been recently proposed in quantum information theory. Near-horizon regions of large-mass black holes, a POVM measurement defined by Eqs. (5) and (6) is performed for quantum fluctuation of N massless scalar fields in the Hartle-Hawking state, which is effectively described by the Minkowski vacuum state $|0\rangle$. Then, we obtain N -bit information about the fluctuation denoted by D_k . During the measurement, wave packets of the fields with positive energy given by Eq. (16) form and then fall into the black hole. The amount of excitation energy is independent of the measurement results. After absorption of the wave packets and increase in the black hole mass, a D_k -dependent local operation of the N fields given by Eq. (23) is performed outside the horizon. Then, accompanying the extraction of positive energy from the fluctuation by the operation, wave packets of the fields with negative energy given by Eq. (32) form and then fall into the black hole. Because both the mass and the entropy of the black hole decrease due to absorption of the negative flux, this QET process is analogous to the Hawking process generated by pair creation of excitations with positive and negative energy, although the energy extracted by QET does not show the thermal properties.

In general, black holes swallow various kinds of information from falling matter, which increases the area of the horizon. Because falling matter with the same mass, charge, and angular momentum generates the same geometry in classical theory, classical black holes forget the information after the shifted horizon stabilizes. Meanwhile, many physicists believe, respecting the unitarity of theory, that quantum black holes do not forget the information and that the detailed memory is stored in some quantum mechanical ways. However, though many efforts to understand the quantum memory storage mechanism have been made so far, the complete resolution remains elusive. Even in the case of large-mass static black holes, the question as to where the information is stored in the black-hole spacetimes has not been completely resolved. The information might be stored inside the horizons, on the horizons, or even outside the horizons. This is the so-called black

hole entropy problem. From the results of this paper, we can conclude that some memories of absorbed quantum matter remains outside the horizon for a while, even after the shifted horizons are completely settled. The reason is following. If the unitary operation in Eq. (23) corresponding to a different N -bit sequence $D_{k' \neq k}$ is performed after the black-hole absorption of wave packets with information D_k , the amount of energy extracted by QET always decreases. Any quantum operation U_b with a wrong bit number b does not generate negative energy; instead, it generates positive energy of the field. Hence, this execution of U_b needs work. The amount of energy is $E'_B = 3\eta^2/4\xi (> 0)$. Figure 5 shows a schematic diagram for the case of a wrong N -bit sequence $D_{k'} = (011 \cdots 1)$. Thus, if the wrong bit number per N exceeds $1/4$, there is no energy gain from the black hole on average by the QET process for the N fields. In an extreme case, a random choice of U_b yields $\eta^2/4\xi$ energy input to the field on average. The black hole returns the maximum amount of energy by QET if the unitary operation obeying the correct D_k is performed on the N fields. Therefore, the black hole remembers which D_k state was absorbed. This implies that the black holes absorbing quantum states with different D_k are physically different even though their black-hole geometries are the same outside the horizons. In this case, the information of D_k is imprinted in the quantum fluctuation of the fields outside the horizon. In fact, in the region out of the horizon, the one-point function of $\hat{\Pi}_+$ depend on the measurement result D_k as seen in Eq. (22). Though this observation is of much interest, these memories may be, in a precise sense, not black hole hairs contributing to the black-hole entropy. This is because the memories are not stored eternally, but may decay in time T as suggested by Eq. (31). This loss of memories over time suggests that the black-hole QET can be used as a probe for the thermal relaxation process of the black hole to a true quantum thermal equilibrium state.

Before closing this discussion, I would like to add a comment about the Unruh effect in the Rindler spacetime [16] [17]. If we consider a detector coupled with quantum fluctuation of fields and uniformly accelerated in the vacuum, we observe thermal excitation in the detector as the Unruh effect. This effect is artificially caused by measurements and not a spontaneous effect. The artificially induced effect is similar to the QET method described in this paper. However, the QET method is completely different from the Unruh effect. The detector of our POVM measurement is coupled with quantum fluctuation for a very short duration, though detectors used for observation of the Unruh effect must be coupled with fluctuation over a very long period

of time and should be simultaneously accelerated uniformly. The detector for QET is not accelerated at all. Moreover, the measurement results read out of the detector are essential for QET. We give a quantum feedback to the field based on the measurement result. Because of the dependence on the measurement result, the local operation given in Eq. (23) can generate negative energy on average of the field with extraction of positive energy outside the horizon. However, the measurement result in the Unruh effect is not reused to give any feedback to the field. This is a quite different point from our model. In the reference [17], Unruh and Wald also discussed a correlation of response of a particle detector with energy generated in a causally disconnected region from the detector of the Rindler spacetime. This causally disconnected region corresponds to inside an eternal black hole when the detector is located outside the horizon. Interestingly, positive energy density is induced in the causally disconnected region when the particle detector outside the horizon is observed in the excited state. When the detector is observed in the ground state, negative energy is induced in the region such that the average energy density in the causally disconnected region remains zero which is equal to the vacuum-state value. However, it should be stressed that the energy induced by the detector is located inside the horizon and cannot be used for a person who stays outside the horizon. Moreover, the average energy (mass) of the black hole does not change at all after the outside-horizon measurement. This feature is completely different from that of our QET model. In the QET model, the person outside the horizon can effectively get positive energy from the black hole via quantum fluctuation of the field outside the horizon. The negative energy given by Eq. (32) is generated outside the horizon by the measurement and falls into the black hole so as to decrease the black hole energy.

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Figure Captions

Figure 1: Spacetime diagram of the QET process in the near-horizon region. The future horizon stays at $x^- = 0$. At $t = 0$, quantum fluctuation of \hat{f} in the spatial region $[x_1, x_2]$ is indirectly measured by a spin probe. A wave packet with positive energy E_A is generated by the measurement device; thereafter, it falls into the future horizon. At $t = T$, the unitary operation U_b , which is dependent on the measurement result b , is performed; this operation generates a wave packet with negative energy $-|E_B|$, which falls into the horizon by extracting positive energy from the field.

Figure 2: Spacetime diagram of the horizon shift in the X coordinates. The initial horizon H_I stays at $X^- = 0$. By absorbing the positive energy flux generated by the measurement, the horizon moves outwards. H_F denotes the shifted horizon. After the absorption of negative energy flux generated by U_b , the horizon recedes. The shifted horizon is denoted by H_{QET} . The positions of the horizons are fixed soon after the wave packets are swallowed. The argument does not change at all if the falling energy flux interacts with matter within the black hole. In the diagram, right-moving matter, which is represented by dashed lines, collides with the falling positive-energy flux inside the horizon.

Figure 3: Schematic diagram of the measurement process of QET. The N cylinders in the figure represent the measurement devices used to detect the quantum fluctuation of the N fields. In this figure, the N -bit output of the devices is $D_k = (101 \cdots 0)$. Each detector creates a wave packet with positive energy E_A that is independent of b . The wave packets are absorbed by the horizon, which is represented by the shaded region to the left.

Figure 4: Schematic diagram of a unitary operation process that is dependent on the measurement result. The N cubes in the figure represent the operation devices of the unitary operation for the N fields. After the operation, a wave packet with negative energy $-|E_B|$ is emitted from each device and is absorbed by the horizon. Before this, the horizon has already absorbed the wave packets corresponding to $D_k = (101 \cdots 0)$ that are generated by the measurement. Local energy conservation assures extraction of positive energy $+|E_B|$ from each field by the device.

Figure 5: Schematic diagram of a wrong operation on the quantum fields. In the figure, a unitary operation corresponding to an N -bit data $(011 \cdots 1)$ that is different from the correct data $(101 \cdots 0)$ is performed. Devices with wrong bit numbers do not generate negative energy flux, but generate positive energy flux with $+|E'_B| = 3\eta^2/4\xi$. When the wrong bit number per N exceeds $1/4$, there is no energy gain from the black holes on average by the QET process.

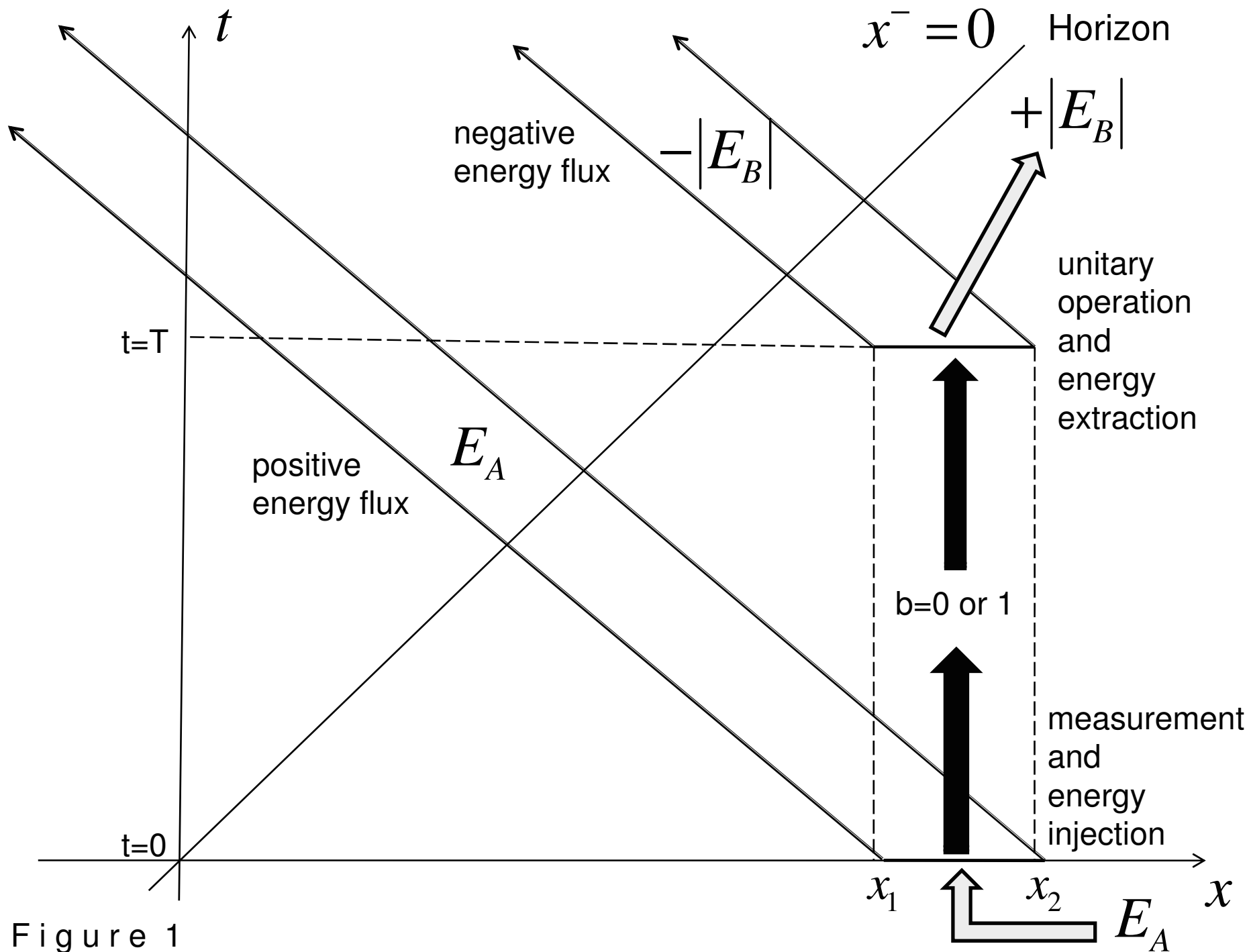


Figure 1

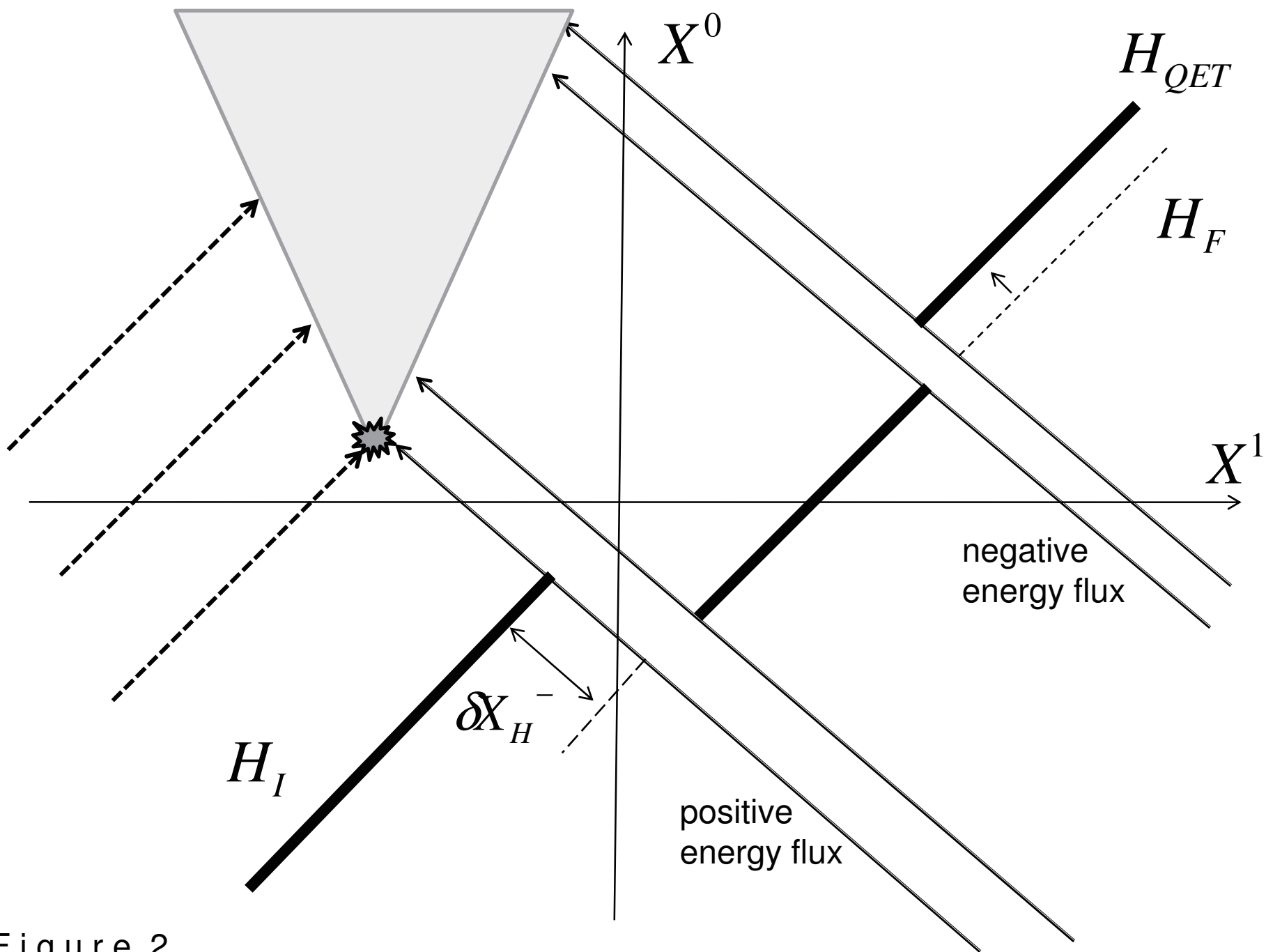
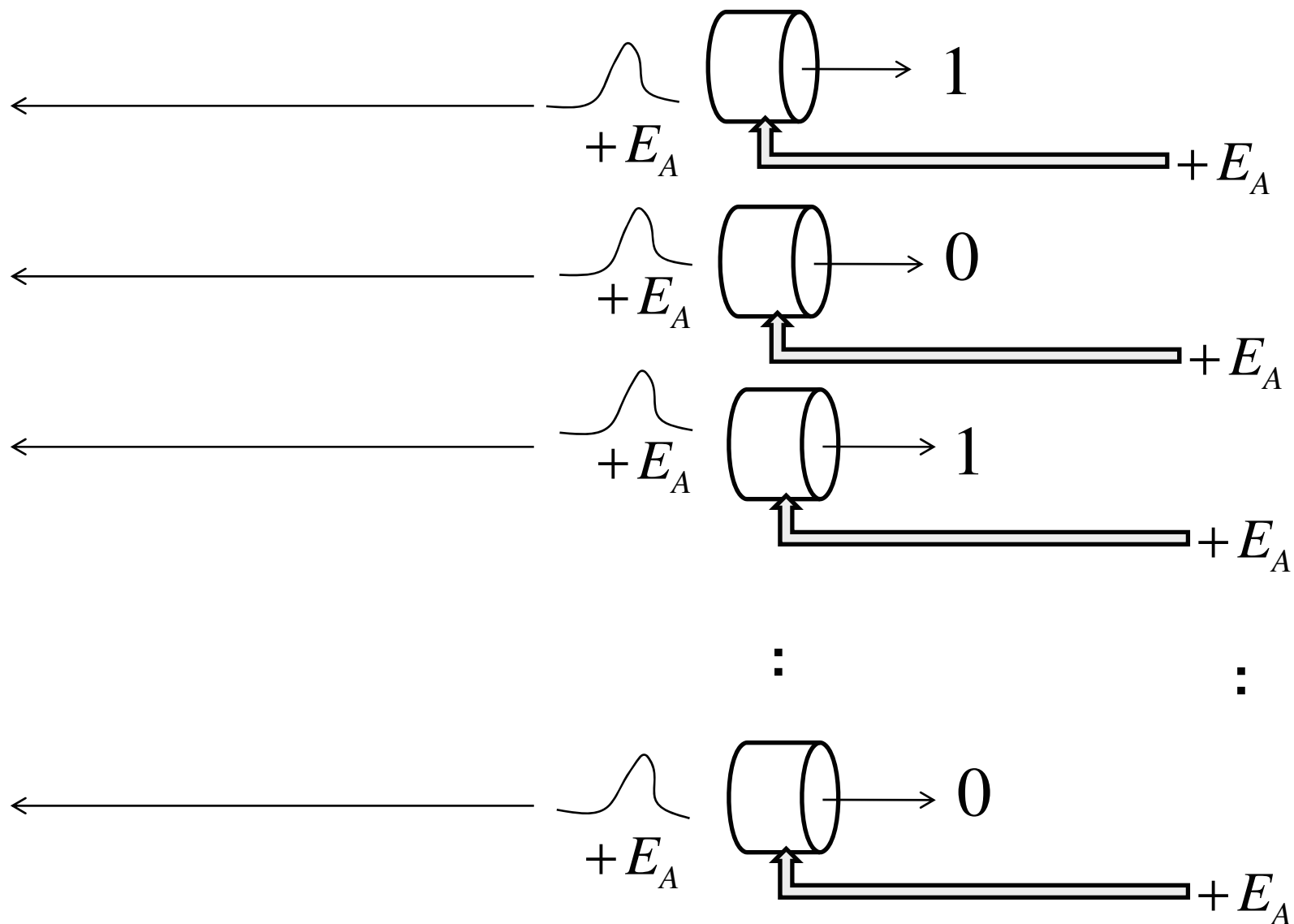


Figure 2

$$D_k = (101..0)$$



Horizon

Figure 3

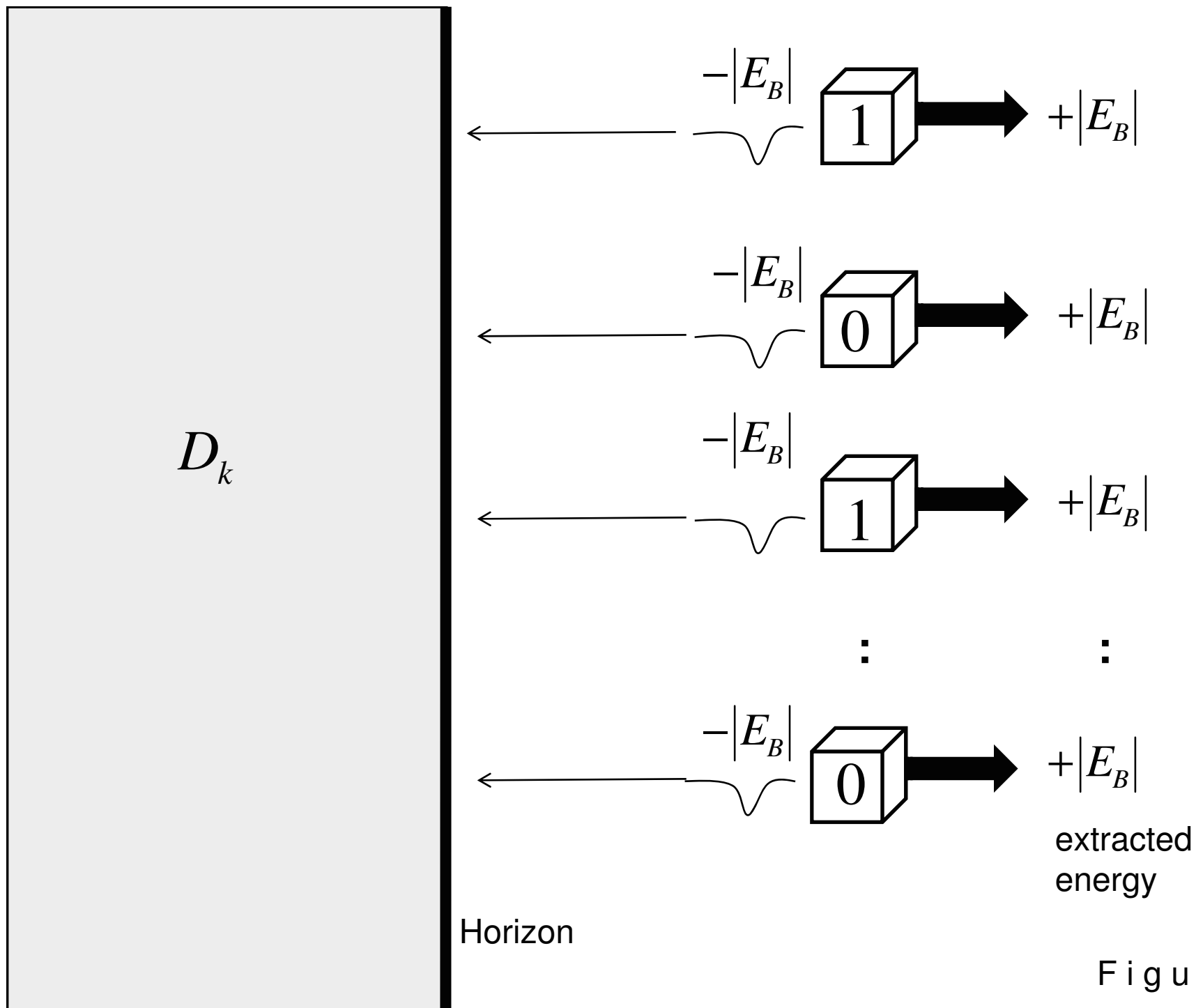


Figure 4

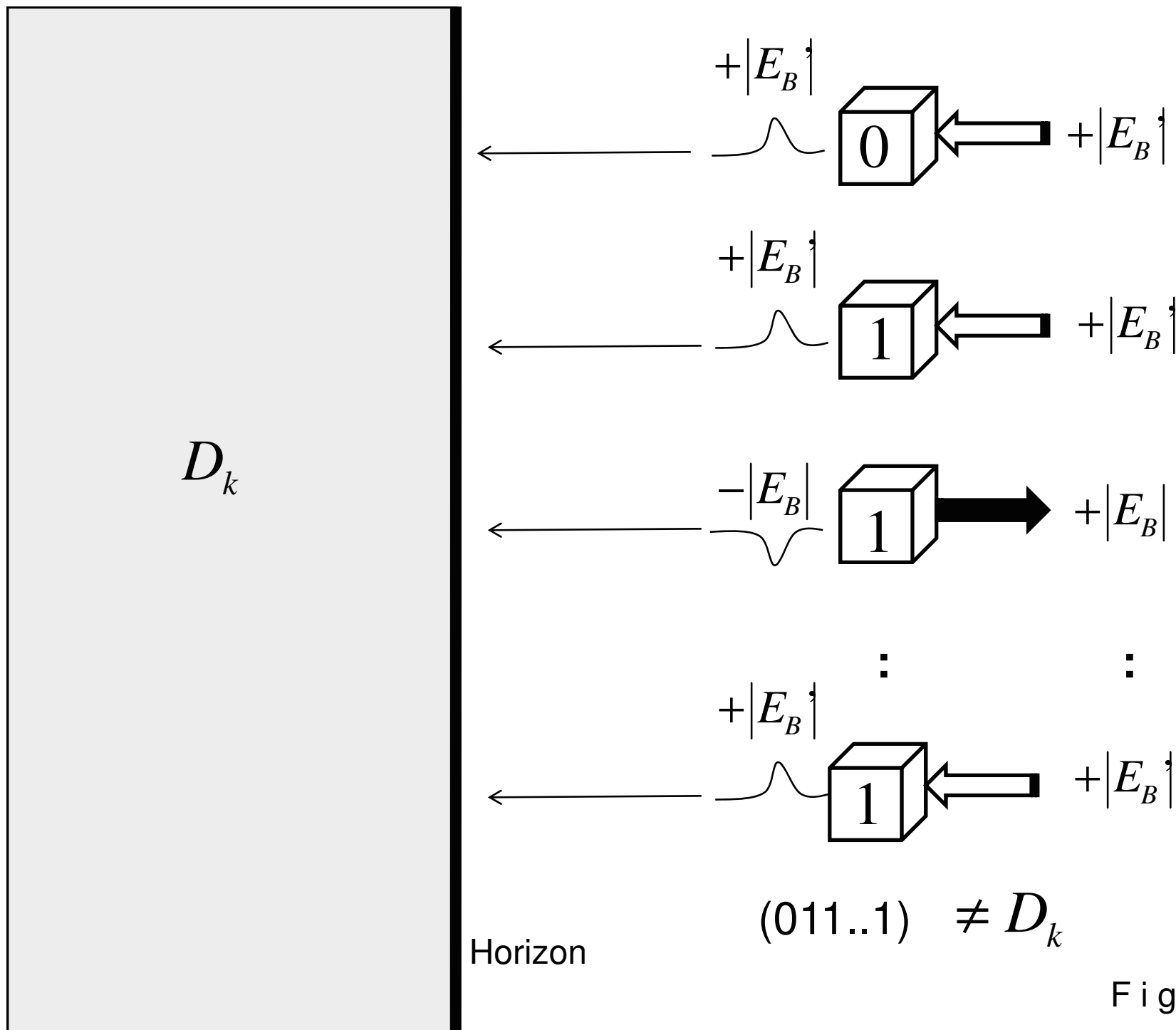


Figure 5